On-line learning
Where are we so far?

Antoine Cornuéjols

MMIP, AgroParisTech, Paris

October 9th, 2008

“Incremental learning”: a new topic?

The first learning algorithms were all incremental:

- Perceptron [Rosenblatt, 1957-1962]
- CHECKER [Samuel, 1959]
- ARCH [Winston, 1970]
- Version Space [Mitchell, 1978, 1982], ...

However, **most existing learning algorithms are not!**

- C4.5 / Regression trees / ...
- SVM / Neural Networks / ...
- ILP systems / Grammatical inference / ...
- ...

Outline

Outline

Un exemple

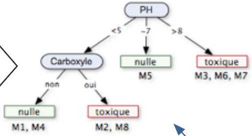
Attributs

Étiquettes

	#Cycles	Masse	PH	Carboxyle	Activité
M1	1	faible	<5	non	nulle
M2	2	moyen	<5	oui	toxique
M3	0	moyen	>8	oui	toxique
M4	0	moyen	<5	non	nulle
M5	1	lourd	~7	non	nulle
M6	2	lourd	>8	non	toxique
M7	1	lourd	>8	non	toxique
M8	0	faible	<5	oui	toxique

Exemple

Valeurs des attributs



R1: Si pH > 8
alors toxique
R2 : Si pH < 5 & Carboxyle = oui
alors toxique
....

Concept
ou
hypothèse

The standard setting

Learning algorithms geared to the analysis of large data bases

- **Stationary and identical distribution** for learning and test
- **i.i.d. assumption** (independently and identically distributed)

$$\boxed{\mathbf{P}_{\mathcal{Y}}} \xrightarrow{\;y\;} \boxed{\mathbf{P}_{\mathcal{X}|\mathcal{Y}}} \xrightarrow{\;<\boldsymbol{x}, y>\;}$$

Figure: Generative process for the examples.

Almost correct prediction (most of the time) (PAC)

$$L(h) \;=\; \mathbf{P}_{\mathcal{X}\mathcal{Y}}\{h(\boldsymbol{x}) \neq y\}$$

The standard setting

Optimizing the expected risk

Real risk: expected loss

$$R(h) \;=\; \mathbb{E}[\ell(h(\boldsymbol{x}), y)] \;=\; \int_{\boldsymbol{x}\in\mathcal{X}, y\in\mathcal{Y}} \ell(h(\boldsymbol{x}), y) \; \mathbf{P}_{\mathcal{X}\mathcal{Y}} \; d(\boldsymbol{x}, y)$$

But $\mathbf{P}_{\mathcal{X}\mathcal{Y}}$ is unknown, then use: $\mathcal{S}_m = \{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_m, y_m)\} \in (\mathcal{X} \times \mathcal{Y})^m$

Empirical risk Minimization

$$\hat{h} \;=\; \underset{h\in\mathcal{H}}{\text{ArgMin}} \big[R_m(h)\big] \;=\; \underset{h\in\mathcal{H}}{\text{ArgMin}} \left[\frac{1}{m} \sum_{i=1}^{m} \ell(h(\boldsymbol{x}_i), y_i)\right]$$

Un dilemme fondamental
Le compromis biais-variance

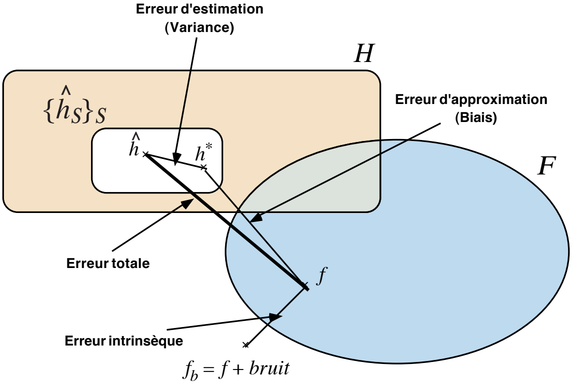


FIG.: Les différents types d'erreurs.

The standard setting
Optimizing the expected risk

Real risk: expected loss

$$R(h) \;=\; \mathbb{E}[\ell(h(\boldsymbol{x}), y)] \;=\; \int_{\boldsymbol{x}\in\mathcal{X}, y\in\mathcal{Y}} \ell(h(\boldsymbol{x}), y)\; \mathbf{P}_{\mathcal{X}\mathcal{Y}}\; d(\boldsymbol{x}, y)$$

But $\mathbf{P}_{\mathcal{X}\mathcal{Y}}$ is unknown, then use: $\mathcal{S}_m = \{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_m, y_m)\} \in (\mathcal{X} \times \mathcal{Y})^m$

Empirical risk Minimization

$$\hat{h} \;=\; \underset{h\in\mathcal{H}}{\mathrm{ArgMin}}\big[R_m(h)\big] + \text{Reg}\big] \;=\; \underset{h\in\mathcal{H}}{\mathrm{ArgMin}}\left[\frac{1}{m}\sum_{i=1}^{m} \ell(h(\boldsymbol{x}_i), y_i)\right] + \lambda\,\text{Capacity}(\mathcal{H})\big]$$

1. All examples are equal: **no forgetting**
2. Commutative criterion: **no information from the sequence**

Capacité de $\mathcal{H}$

Qualité de l'estimation fonction de $\mathcal{H}$

$$|R(h) - R_{\mathtt{Emp}}(h)| \ \leq_P \ fct(\mathtt{diversit\acute{e}}_{\mathcal{H}}, m)$$

- Dimension de Vapnik-Chervonenkis
- Complexité de Rademacher
- BIC, AIC
- ...

Qualité de l'estimation fonction de l'expression de h

$$|R(h) - R_{\mathtt{Emp}}(h)| \ \leq_P \ fct(\mathtt{complexit\acute{e}}(h), m)$$

- Nombre de connexions
- Nombre de points de support
- ...

Choix de $\mathcal{H}$

Types d'espaces d'hypothèses

Modèles génératifs

Demandent $p(\boldsymbol{x}|\mathcal{C}_k)$ et $p(\mathcal{C}_k)$

Fonctions de décision

- Fonctions composées de fonctions de base
- Méthodes à Noyaux (*Kernel methods*)

Approches constructives

- Spécialisation dans l'espace $\mathcal{X}$
- Modèles hiérarchiques ou “profonds” (*Deep models*)

Choix de $\mathcal{H}$
Modèles génératifs

1. **Estimer** $p(\boldsymbol{x}|\mathcal{C}_k)$ (et $p(\mathcal{C}_k)$) $\forall \mathcal{C}_k$

   par le ***Principe du Maximum de Vraisemblance*** (MLE)

2. **Décider**, en utilisant le *théorème de Bayes* :

$$p(\mathcal{C}_k|\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\mathcal{C}_k)\, p(\mathcal{C}_k)}{p(\boldsymbol{x})}$$

avec $$p(\boldsymbol{x}) = \sum_k p(\boldsymbol{x}|\mathcal{C}_k)\, p(\mathcal{C}_k)$$

Exemple : Deux classes supposées gaussiennes de mêmes matrices de covariance

$$p(\boldsymbol{x}|\mathcal{C}_k) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma|^{1/2}} \exp\{-\frac{1}{2}(\boldsymbol{x}-\mu_k)^\top \Sigma^{-1} (\boldsymbol{x}-\mu_k)\}$$

Choix de $\mathcal{H}$

Modèles probabilistes discriminatifs

- **Estimer directement** $p(\mathcal{C}_k|\boldsymbol{x})$ en utilisant un modèle paramétré

Exemple : Deux classes et régression logistique

$$p(\mathcal{C}_1|\boldsymbol{x}) = \frac{1}{1+\exp(\boldsymbol{w}^\top \boldsymbol{x})} = \sigma(\boldsymbol{w}^\top \boldsymbol{x})$$

1. Fonction de vraisemblance : $p(\boldsymbol{y}|\boldsymbol{w}) = \Pi_{i=1}^m \sigma(\boldsymbol{w}^\top \boldsymbol{x}_i)^{y_i} \left\{1-\sigma(\boldsymbol{w}^\top \boldsymbol{x}_i)\right\}^{(1-y_i)}$
2. Fonction d'erreur :
$E(\boldsymbol{w}) = -\ln p(\boldsymbol{y}|\boldsymbol{w}) = -\sum_{i=1}^m \{y_i \ln \sigma(\boldsymbol{w}^\top \boldsymbol{x}_i)_i + (1-y_i)\ln(1-\sigma(\boldsymbol{w}^\top \boldsymbol{x}_i)_i)\}$
3. Optimisation par ***descente de gradient***

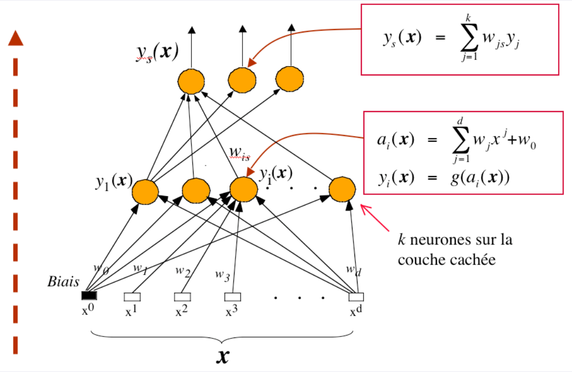
Moins de paramètres à estimer.

Choix de $\mathcal{H}$

Fonctions de décision à base de dictionnaire

- $h(\boldsymbol{x}, \boldsymbol{w}) \;=\; \sum_{i=1}^{n} w_i \, g_i(\boldsymbol{x}) + w_0$
- où les $g_i(\boldsymbol{x})$ sont des ***fonctions de base***

Exemple : Perceptron multi-couches



Choix de $\mathcal{H}$

Fonctions de décision par noyaux

- $h(\boldsymbol{x}) \;=\; \sum_{i \text{ “critiques”}} \alpha_i\, y_i\, K(\boldsymbol{x}, \boldsymbol{x}_i) + \alpha_0$
- où les $K_i(\cdot,\cdot)$ sont des ***fonctions noyaux***

$$K_G(\boldsymbol{x}, \boldsymbol{x}_i) = \exp\left(-\frac{||\boldsymbol{x}, \boldsymbol{x}_i||^2}{2\,\sigma^2}\right)$$

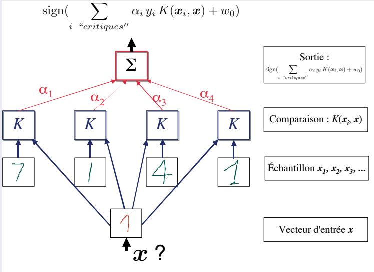
$$K_L(\boldsymbol{x}, \boldsymbol{x}_i) = \boldsymbol{x}^\top \boldsymbol{x}_i$$

$$K_{Poly1}(\boldsymbol{x}, \boldsymbol{x}_i) = (\boldsymbol{x}^\top \boldsymbol{x}_i)^d$$

$$K_{Poly2}(\boldsymbol{x}, \boldsymbol{x}_i) = (\boldsymbol{x}^\top \boldsymbol{x}_i + c)^d$$

$$K_{sig}(\boldsymbol{x}, \boldsymbol{x}_i) = \tanh(\kappa \boldsymbol{x}^\top \boldsymbol{x}_i + \theta)$$

Exemple : Séparateurs à Vastes Marges (SVM)

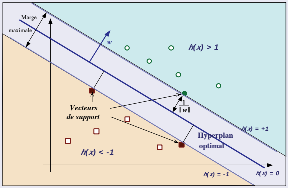


Choix de $\mathcal{H}$

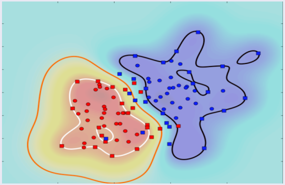
Fonctions de décision par noyaux : Les SVM

$$h^*(\mathbf{x}) \;=\; (\boldsymbol{w}^\star\ \mathbf{x}) + w_0^\star \;=\; \sum_{i=1}^{m} \alpha_i^\star\, u_i\,.\,\langle \phi(\mathbf{x}_i)\,,\,\phi(\mathbf{x})\rangle + w_0^\star$$

SVM : espace des redescripteurs $\Phi(\mathcal{X})$



SVM : espace initial $\mathcal{X}$

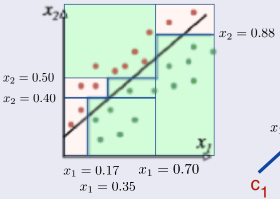


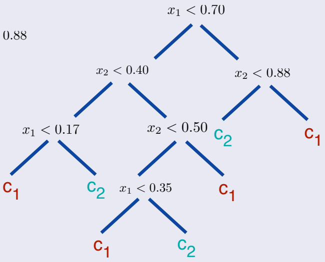
Choix de $\mathcal{H}$

Fonctions de décision par spécialisation

$$
\begin{aligned}
h_{\mathcal{X}} &= h_{\mathcal{X}_1} \times h_{\mathcal{X}_2} \times \ldots \times h_{\mathcal{X}_n} \\
\mathcal{X} &= h_{\mathcal{X}_1} \cup h_{\mathcal{X}_2} \cup \ldots \cup h_{\mathcal{X}_n}
\end{aligned}
$$

Exemple : Arbres de décisions

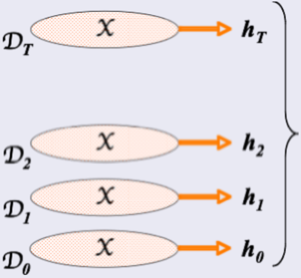




Choix de $\mathcal{H}$

Fonctions de décision par vote

Exemple : Boosting



$$h(\boldsymbol{x}) \;=\; \text{sign}\left[\sum_{t=0}^{T} \alpha_t \cdot h_t(\boldsymbol{x})\right]$$

Key idea n°1:

Essential character of the inductive criterion

- **Expresses the problem:**
  - Cost of misclassification
  - Global measure of performance taken as a substitute for the real one
- Allows us to **analyze** the conditions for a successful induction
- Did **motivate** most modern learners (SVM, Boosting, ...)

Le paradigme ... et ses limites

- Lien entre passé et futur :

  distributions $P_{\mathcal{X}}$ et $P_{\mathcal{Y}|\mathcal{X}}$ supposées stationnaires

- Données i.i.d.

Why go over on-line learning one more time ... ?
... now ?

Increasing interest

- Workshop on *Dynamically changing domains: Theory revision and context dependence issues* (ECML-97)
- Intelligent Data Analysis journal *Special issue on Incremental Learning Systems Capable of Dealing with Concept Drift*, Vol.8, N0.3, 2004.
- Workshop *First International Workshop on Knowledge Discovery in Data Streams*, ECML-04.
- ACM Symposium on Applied Computing (SAC-2006) *Special Track on Data Streams*.
- Workshop *Second International Workshop on Knowledge Discovery in Data Streams*, ECML-05.
- Workshop à NIPS-2006 : *Learning when test and training inputs have different distributions*

Measuring and controlling a world in movement

New types of data

- Data are made available through *unlimited streams* that continuously flow, possibly at high-speed
- The underlying *regularities may evolve over time* rather than be stationary
- The data is now often *spatially as well as time situated*

The data can **no longer** be considered as ***independent and identically distributed***

On-line learning: why bother?

A wealth of new applications

1. **Limited resources:**
   - Learning from very large data bases (e.g. Telecoms: millions of examples ; EGEE: billions of examples, ...)
2. **"Anytime" constraints**: Data streaming
3. **Covariate shift**: stationary target concept but changing distribution
4. **Active learning**
5. **Concept drift**
6. **Transfer learning** from one task to another
7. **Tutored learning** with a professor

Outline

Empirical incremental learning

Lots of empirical and heuristic techniques

to solve computational and on-line issues

- **Reduce computational cost**
- **Classification**
  - k-nearest neighbors
  - Decision Trees

- **Issues raised**

Stochastic gradient vs. total gradient
Total gradient

$$\hat{h} \;=\; \underset{h \in \mathcal{H}}{\text{ArgMin}}\, R_m(h) \;=\; \underset{h \in \mathcal{H}}{\text{ArgMin}}\, \frac{1}{m} \sum_{i=1}^{m} \ell(h(\boldsymbol{x}_i), y_i)$$

Total gradient

$$\begin{aligned} h_t &= h_{t-1} - \Phi_t \frac{\partial R_m(h_{t-1})}{\partial h} \\ &= h_{t-1} - \Phi_t \frac{1}{m} \frac{\partial}{\partial h} \sum_{i=1}^{m} (h_{t-1}(\boldsymbol{x}_i), y_i) \end{aligned}$$

Linear convergence towards the optimum $\hat{h}$ de $R_m(h)$: $(h_t - \hat{h})^2$ converges as e^{-t}.

BLC05 L. Bottou et Y. Le Cun (2005) *"On-line learning for very large datasets"* Applied Stochastic Models in Business and Industry, 21(2):137-151, 2005.

Stochastic gradient vs. total gradient
Stochastic gradient

$$\hat{h} \;=\; \underset{h \in \mathcal{H}}{\text{ArgMin}}\, R_m(h) \;=\; \underset{h \in \mathcal{H}}{\text{ArgMin}}\, \frac{1}{m} \sum_{i=1}^{m} \ell(h(\boldsymbol{x}_i), y_i)$$

Stochastic gradient

$$h_t \;=\; h_{t-1} - \frac{1}{t}\, \Phi_t\, \frac{\partial L}{\partial h}(h_{t-1}(\boldsymbol{x}_t), y_t)$$

- Converges slowly towards a local optimum of $R_m(h)$: $(h_t - \hat{h})^2$ converges as $\frac{1}{t}$.
- In fact, converges quickly towards the region of the optimum but slowly then because of the noisy (stochastic) gradient.

Much **simpler** than batch

Stochastic gradient vs. total gradient

Computational complexity

- Batch
  - Store N examples
  - Gradient in $\mathcal{O}(N)$ operations
- On-line
  - Must memorize the “sufficient past” (in h_t)
  - Gradient in $\mathcal{O}(1)$ operations

Approximation

- Batch
  - Converges towards $h* = \mathrm{ArgMin}_{h \in \mathcal{H}} R(h)$ with approximation $\mathcal{O}(1/t)$
- On-line
  - Converges towards $h* = \mathrm{ArgMin}_{h \in \mathcal{H}} R(h)$ with approximation $\mathcal{O}(1/t)$
  - But **can consider more examples !!**
    ($\mathcal{O}(N \log N)$ instead of N for batch)

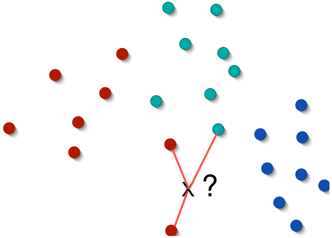
Key idea n°2:

On-line learning is simpler (less costly)

... which can lead to better learning performance!

Incremental learning
Illustration

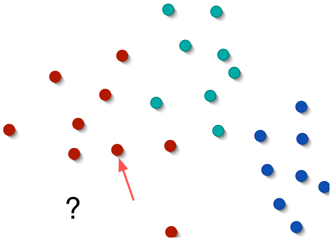
Nearest neighbors



- Simple algorithm ("*lazy learning*")
- Order independent
- But growing computational cost (time and space): $\mathcal{O}(m)$

Incremental learning
Illustration

Nearest neighbors (2)
with limited memory

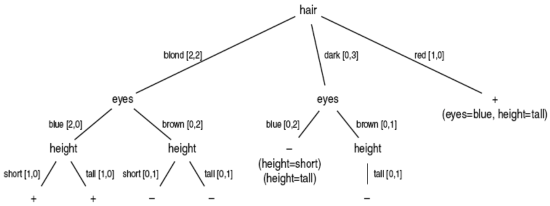


- Selection of “prototypes”
    - Eliminate the outlier
    - Eliminate the most ancien
    - Compute and keep center of gravity with the closest point
    - ...
- Order dependent

Incremental learning
Illustration

Incremental induction of decision trees (ID5R)

class	height	hair	eyes
–	short	blond	brown
–	tall	dark	brown
+	tall	blond	blue
–	tall	dark	blue
–	short	dark	blue
+	tall	red	blue
–	tall	blond	brown
+	short	blond	blue



- Actually memorizes all examples
- Order independent
- But computational time at each step : $\mathcal{O}(m \cdot d \cdot b^d)$

UTG89 Paul Utgoff (1989) *"Incremental Induction of Decision Trees"* Machine Learning Journal, vol.4, No.2, 161-186

Incremental learning
Assessment

Numerous heuristic algorithms

Motivations

- Computational constraints (e.g. constant time)
- Time series

New questions

- What to keep in memory?
- Sequence effects
  - How to reduce them?
  - (How to use the information in the sequence?)

Data streams

A first illustrative example

Paul presents a permutation of the n first integers, one by one, with one integer missing

E.g.: $n = 10$ 3, 7, 6, 8, 5, 1, 9, 2

Carol must identify the missing number.

Caveat : She cannot store all the numbers seen so far (which would take $\mathcal{O}(n \log n)$ digits).

Solution:

- **Carol** adds all the numbers seen so far ($\mathcal{O}(\log n)$ digits and computational cost $\mathcal{O}(\log n)$
- and she subtracts that sum from $\frac{n \cdot (n+1)}{2}$

Total cost : space $\mathcal{O}(\log n)$ digits ; computation $\mathcal{O} \log n$

Data streams

A first illustrative example

What if **Paul** shows all but two numbers?

E.g.: $n = 10$ 3, 7, 6, 5, 1, 9, 2

Solution:

- **Carol** keeps the ***sum*** and ***sum of squares*** of numbers seen so far ($\mathcal{O}(n \log n)$ digits and computational cost $\mathcal{O}(n \log n)$)

- $\sum_{i=1}^{9} x_i = \frac{n \cdot (n+1)}{2} = 9 \cdot 10/2 = 45$
- $\sum_{i=1}^{9} x_i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{9 \cdot 90 \cdot 19}{6} = 285$
- $3 + 7 + 6 + 5 + 1 + 1 + 9 + 2 = 33$
- $9 + 49 + 36 + 25 + 1 + 81 + 4 = 205$
- $x_1 + x_2 = 12$ et $x_1^2 + x_2^2 = 80$
- d'où $x_1 = 4$ et $x_2 = 8$

Data streams
Lessons

Lessons:

The solution relies on keeping **summaries** of the data

Here, the solution is exact. Most of the time, it can only be **approximate**

Data streams

Challenges

- **Not enough space** to store all the stream data
- Hence, impossible to rescan the whole data set
- Need to **adapt to changing data distribution**
- Processing should be **as fast as possible**

Data streaming is an incremental process

(i.e. new iterations are built based on previous ones)

Data streams
Frequently used learners

Desirable properties

- Low runtime complexity
- Inherently incremental

Frequently used learners

- Decision trees (VFDT, adaptive VFDT)
- Rule learners
- SVM (Support Vector Machines)

- Naïve Bayes
- Instance-based learners (e.g. k-nearest neighbors)

Outline

On-line learning: the issues

Computational constraints

Possible in principle with standard (one-shot and i.i.d.) approach, but too costly computationally

→ **Reduce time and space complexity**

- ***Stochastic gradient / incremental learning***

Stationary environment but changing distribution + anytime constraints

Not i.i.d.

→ **Anticipate and take advantage of sequence information**

- ***covariate shift / transductive learning / tracking***

Changing environment

Not i.i.d. + Non stationary

→ **Anticipate and take advantage of sequence information**

- ***concept drift***
- ***transfer learning / tutored learning***

Covariate shift
Definition

Changing $\mathbf{P}_{\mathcal{X}}$

Examples:

- **Non stationary input data** ($\mathbf{P}_{\mathcal{X}}$ changes but not $\mathbf{P}_{\mathcal{Y}|\mathcal{X}}$)
  - Medicine: seasonal variations
  - Spam filtering (adaptation to a new user)
- **Bias in the selection process** in learning
  - Artificially balanced training data (but not in test)
  - Active learning
  - Interpolation vs. extrapolation (in regression)

Covariate shift

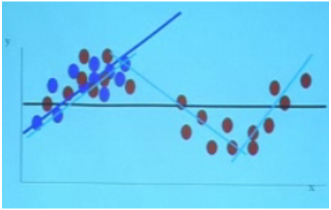
Why is it a problem?

No longer a “direct” link between empirical risk and real risk

Modify the inductive criterion

The performance for the target distribution $\mathbf{P}'_{\mathcal{X}}$ (*generalization*) depends on :

- The performance for $\mathbf{P}_{\mathcal{X}}$ (*learning*)
- The similarity between $\mathbf{P}_{\mathcal{X}}$ and $\mathbf{P}'_{\mathcal{X}}$



Covariate shift

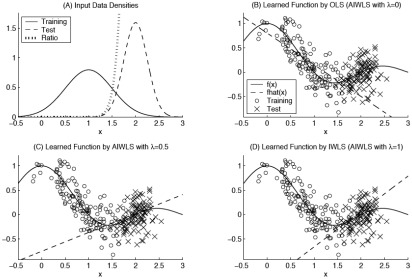
Approaches

“Importance weighted” inductive criterion

Principle : weighting the classical ERM

$$R_{Cov}(h) \; = \; \frac{1}{m} \sum_{i=1}^{m} \left(\frac{\mathbf{P}_{\mathcal{X}'}(\boldsymbol{x}_i)}{\mathbf{P}_{\mathcal{X}}(\boldsymbol{x}_i)} \right)^{\lambda} (h(\boldsymbol{x}_i - y_i)^2$$

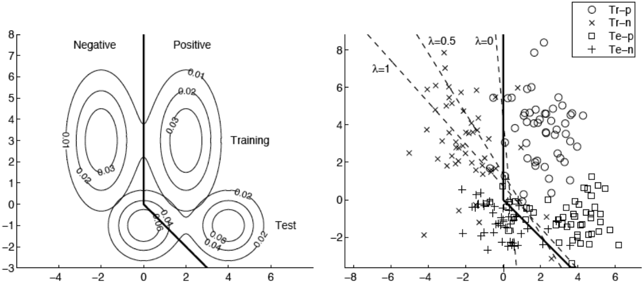
λ controls the stability / consistency (absence of bias)



SKM07 M. Sugiyama and M. Kraudelat and K.-R. Müller (2007) *“Covariate Shift Adaptation by Importance Weighted Cross Validation”* Journal of Machine Learning Research, vol.8: 985-1005.

Covariate shift
Approaches

"Importance weighted" inductive criterion (classification task)



(a) Contours of training and test input densities.

(b) Optimal decision boundary (solid line) and learned boundaries (dashed lines). '○' and '×' denote the positive and negative training samples, while '□' and '+' denote the positive and negative test samples. Note that the test samples are not given in the training phase; they are plotted in the figure for illustration purposes.

SKM07 M. Sugiyama and M. Kraudelat and K.-R. Müller (2007) *"Covariate Shift Adaptation by Importance Weighted Cross Validation"* Journal of Machine Learning Research, vol.8: 985-1005.

Covariate shift
Approaches

"Importance weighted" inductive criterion

How to get $\frac{\mathbf{P}_{\mathcal{X}'}(\boldsymbol{x}_i)}{\mathbf{P}_{\mathcal{X}}(\boldsymbol{x}_i)}$?

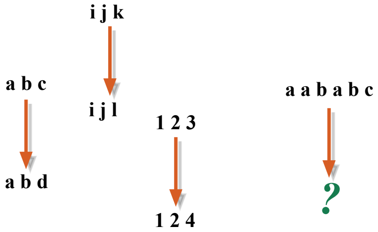
Empirical estimation

Semi-supervised learning

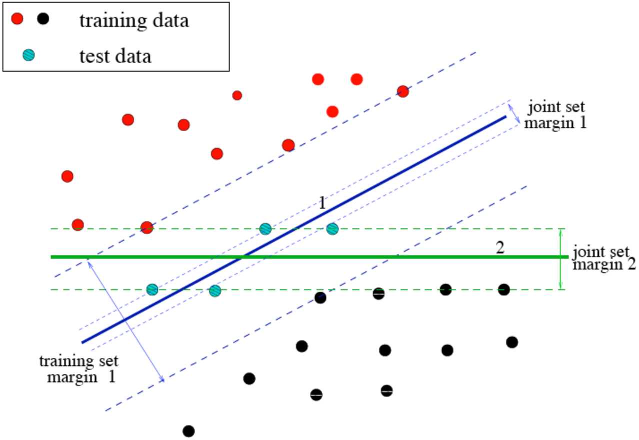
Transduction
Definition

- Given a training set $\mathcal{S}_m = ((\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_m, y_m))$,
  and the knowledge of test data points $\boldsymbol{x}_{m+1}, \ldots, \boldsymbol{x}_{m+k}$
- Identify the best classification vector $y_{m+1}, \ldots, y_{m+k}$ from a given set of possible vectors $Y \in \mathcal{Y}^k$

One is no longer looking for a decision function defined over $\mathcal{X}$!!



Transduction
Methods



On-line learning: the issues

Computational constraints

Possible in principle with standard (one-shot and i.i.d.) approach, but too costly computationally

→ **Reduce time and space complexity**

- ***Stochastic gradient / incremental learning***

Stationary environment but changing distribution + anytime constraints

Not i.i.d.

→ **Anticipate and take advantage of sequence information**

- ***covariate shift / transductive learning / tracking***

Changing environment

Not i.i.d. + Non stationary

→ **Anticipate and take advantage of sequence information**

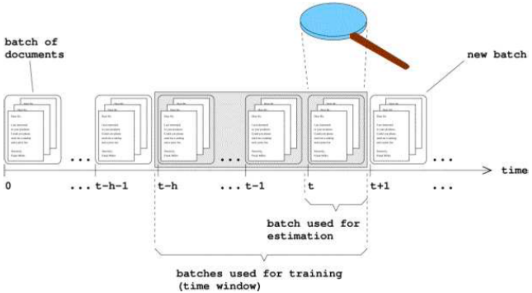
- ***concept drift***
- ***transfer learning / tutored learning***

Concept drift
Definition

Drift of $\mathbf{P}_{\mathcal{X}|\mathcal{Y}}$

Exemples :

- Profiles of customers (purchases function of *income, age, ...*)
- Document filtering function of the interests of the user
- Tracking



Concept drift
Definition

Drift of $\mathbf{P}_{\mathcal{Y}|\mathcal{X}}$

- Profiles of customers (purchases function of *income, age, ...*)
- Document filtering function of the interests of the user

Problems:

- Detecting variations but be robust to noise
- Follow the evolutions but stay robust: Control forgetting

- The oldest the data, the more likely they are obsolete

But:

- The larger the training set, the better the generalization

Heuristic approaches

- **Window based approaches**. *Problem: control their size*
- **Weighting the examples** with respect to time. *Problem: control their weights*

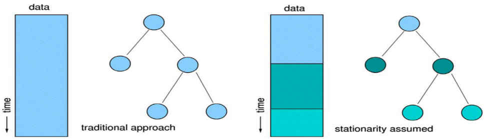
Key idea n°3:

Control the memory of the past

Concept drift

The decision tree approach: VFDT

- Stream Mining: VFDT = Very Fast Decision Tree Learner



- Key question: How much data is needed to safely induce node?
- Use Hoeffding-bound: estimate the nb of examples necessary to acknowledge IG(X_{best}) - IG($X_{\text{2nd best}}$) $> \varepsilon$, with prob. $1 - \delta$

DH00 Domingos, Hulten (2000) *"Mining high-speed data streams"* KDD 2000, 71-80.

Concept drift

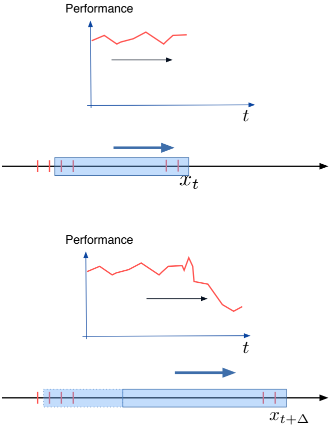
Concept adapting VFDT

- VFDT assumes stationarity
- If no stationarity: node may violates IG(X_{best}) - IG($X_{\text{2nd best}}$) $> \varepsilon$
- If drift suspected:
  - start learning alternative subtree
  - exchange subtree if more accurate than current subtree
- Fast adaptation by forgetting
  - maintain window in memory to correct sufficient statistics at nodes
  - but window of fixed size (forgetting is not adaptive)

HSD01 Hulten, Spencer, Domingos (2001) *"Mining time changing data streams"* KDD 2001, 97-106.

Concept drift
Sliding window approach

Principle:

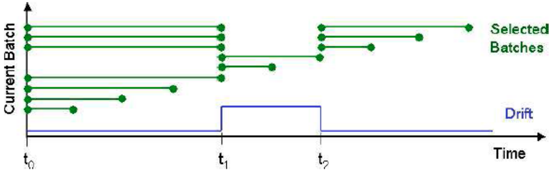


WK96 G. Widmer and M. Kubat (1996) *"Learning in the presence of concept drift ans hidden contexts"* Machine Learning 23: 69–101, 1996.

Concept drift
Sliding window approach

One (among many) method for the selection of windows

- Learn a classifier on the last batch
- Test it on every preceding windows
- Keep the windows where error $< \varepsilon$

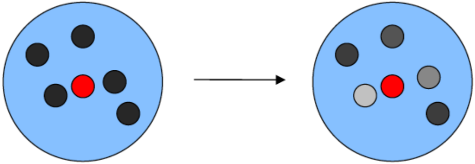


SK M. Scholz and R. Klinkenberg (1996) *"Boosting classifiers for drifting concepts"* Intelligent Data Analysis (IDA) Journal, Volume 11, Number 1, March 2007.

Concept drift
Locally weighted forgetting

Forget *redundant* data first

- All data starts with weight 1
- Weights of k nearest neighbors are adjusted
  - The closer the data to the new sample, the more the weight is decayed
  - If weight drops below some threshold, remove data



Sal97 Salganikoff (1997) *"Tolerating concept and sampling shift in lazy learning using prediction error context switching"* Artificial Intelligence Review, Volume 11, pp.133-155, 1997.

Concept drift
Ensemble methods

- Learn a number of models on different parts of the data
- Weigh classifiers according to recent performance
- If classifier performance degrades, replace it by a new classifier

Concept drift
Ensemble methods

Dynamic weighted majority

- Classifiers in ensemble have initially a weight of 1
- For each new instance:
  - If a ***classifier predicts incorrectly***, reduce its weight
  - If ***weight drops below threshold***, remove classifier
  - If ***ensemble*** then ***predicts incorrectly***, install new classifier
  - Finally, all classifiers are (incrementally) updated by considering new instance

KM03 Kolter, Maloof (2003) *"Dynamic weighted majority: a new ensemble method for tracking concept drift"* ICDM 2003, 123-130.

Concept drift
Ensemble methods

Accuracy weighted majority

- Divide streams into chunks
- Learn new classifier from n such chunks and keep the k top performing classifiers
- Use most recent chunk to estimate expected accuracy of each classifier
- Weigh classifiers in the ensemble by expected accuracy (Results are provably better than by averaging decisions)

WFYH03 Wang, Fan, Yu, Han (2003) *"Mining concept-drifting data streams using ensemble classifiers"* KDD 2003, 226-235.

Concept drift
A Boosting approach

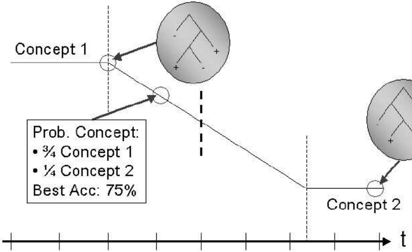


Fig. 4. Continuous concept drift, starting with a pure *Concept 1* and ending with a pure *Concept 2*. In between, the target distribution is a probabilistic mixture. It is optimal to predict *Concept 1* before the dotted line, and *Concept 2*, afterwards.

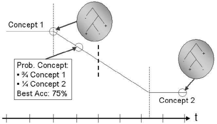
How to learn concept 2 **before** the end of the transition ?

SK M. Scholz and R. Klinkenberg (1996) *"Boosting classifiers for drifting concepts"* Intelligent Data Analysis (IDA) Journal, Volume 11, Number 1, March 2007.

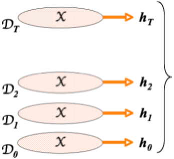
Concept drift
A Boosting approach

Principle :
gradually modify the distribution of the examples

by “substracting” the distribution associated with `concept 1`.



$$\forall \langle \boldsymbol{x}, y \rangle \in \mathcal{X} \times \mathcal{Y} : P_{D'}(\boldsymbol{x}, y) = P_D(\boldsymbol{x}, y) \cdot \frac{P_D[h_{t-1}(\boldsymbol{x}) = \hat{y}] \cdot P_D[y = y*]}{P_D[h_{t-1}(\boldsymbol{x}) = \hat{y}, y = y*]}$$



$$h(\boldsymbol{x}) = \operatorname{sign}\left[\sum_{t=0}^{T} \alpha_t \cdot h_t(\boldsymbol{x})\right]$$

Concept drift
Assessment

Heuristics

- Efficient in their respective application domains
- Require a fine tuning
- Not easily transferable to other domains
- Lack theoretical fundations

Theoretical analyses

What are the conditions for PAC learning with an error of ε ?

- Depends upon $d_{\mathcal{H}}$ and the speed of the drift v (measured as the prob. that 2 subsequent concepts disagree on a randomly drawn example)
- Possible if $v = \mathcal{O}(\varepsilon^2 / d_{\mathcal{H}}^2 \ln \frac{1}{\varepsilon})$ (usually impractically large)
- Rk: adversary protocol

HL94 D. P. Helmbold and P. M. Long (1994) *"Tracking drifting concepts by minimizing disagreements"* Machine Learning 14 (1): 27–45, 1994.

A theoretical approach to on-line learning

A weak framework:

- No assumption about the generative process of the examples

Inductive criterion

- No more notion of risk
- Comparison *a posteriori* to the performance of a set of N "experts"

Learning algorithms

- Maintain a vector of weights on the expert advices

$$\hat{p}_t = \frac{\sum_{i=1}^{N} w_{i,t-1} p_{i,t}}{\sum_{i=1}^{N} w_{i,t-1}}$$

- The weights are function of the *regret* of each expert.

But does not take into account the information in the data sequence

The theoretical viewpoint on on-line learning

Questions ...

Do we have answers to these questions?

- Do we have theoretical guarantees about the performance of usual on-line learning systems (NNs, SVM, ID5, ...)?
  - NO!
- Do we have a satisfactory inductive criterion to replace “Empirical Risk Minimization”?
  - NO!
- Are we able to predict sequence effects?
  - NO!

Concept drift
... further issues

Desirable to:

- *Recognize and treat recurring contexts*
  - E.g. seasonal variations
  - $\rightarrow$ quickly recover old models if appropriate
- *Provide insights about change*
  - **Trends**, “Second derivative"
  - **Interpretability**: “What has changed and how?"
  - By comparison with the last model(s): **discover *interesting* new knowledge**

Useful properties:

- **keeping models**
- Having **a model for change** vs. a model for current context

Concept drift
... further issues

References:

CMM05 Catania, Maddalena, Mazza (2005) *"PSYCHO: A prototype system for pattern management"* Proc. Int. Conf. on Very Large Data Bases, 2005, 1346-1349.

GGR99 Ganti, Gehrke, Ramakrishnan (2005) *"A framework for measuring changes in data characteristics"* Proc. 18th Symp. on Principles of Database Systems, 1999, 126-137.

GGR01 Ganti, Gehrke, Ramakrishnan (2005) *"DEMON: Mining and monotoring evolving data"* IEEE Trans. Knowledge Data Eng., 2001, 13, 50-63.

HB07 Höppner, Bötcher (2007) *"Matching partitions over time to reliably capture local clusters in noisy domains"* Proc. Int. Conf. Principles and Practice of Knowledge Discovery in Databases PKDD (2007), 479-486.

LT08 Liu, Tuzhilin (2008) *"Managing large collections of data mining models"* Communications of the ACM, 2008, 51, 85-89.

MZ05 Mei, Zhai (2005) *"Discovering evolutionary theme patterns from text: an exploration of temporal text mining"* Proc. of KDD 2005, 198-207.

WZFY03 Wang, Zhou, Fu, Yu (2003) *"Mining changes of classification by correspondence tracing"* SIAM Int. Conf. on Data Mining, 2003, 95-106.

Key idea n°4:

Focus on the changes themselves

... and not only on models of the current context

Outline

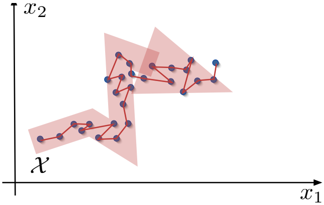
Tracking
Motivation

In a lot of natural settings:

- Data comes *sequentially*
- *Temporal consistency*: consecutive data points come from "similar" distribution: not i.i.d.

This enables:

- Powerful learning
- with limited resources (time + memory)

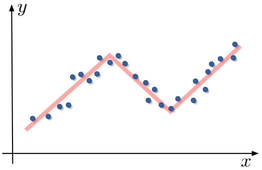


SKS:07 R. Sutton and A. Koop and D. Silver (2007) *"On the role of tracking in stationary environments"* (ICML-07) Proceedings of the 24th international conference on Machine learning, ACM, pp.871-878, 2007.

Tracking
Definition

Assumptions:

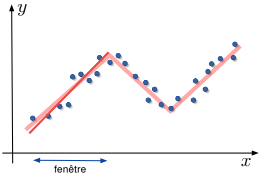
- Data streams
- *Temporal consistency*: consecutive data points come from "similar" distribution: not i.i.d.
- Limited resources: Restricted hypothesis space $\mathcal{H}$



"Local" learning

and local prediction :

$$\begin{aligned} L_t &= \ell(h_t(\mathbf{x}_t), y_t) \\ &= \ell(h_t(\mathbf{x}_t), f(x_t, \theta_t)) \end{aligned}$$



SKS:07 R. Sutton and A. Koop and D. Silver (2007) *"On the role of tracking in stationary environments"* (ICML-07) Proceedings of the 24th international conference on Machine learning, ACM, pp.871-878, 2007.

Tracking
Characteristics

If “temporal consistency” holds ...

... enormous advantage for learning

1. **Less computational cost**
   - *Time*: take into account fewer examples at each time step
   - *Space*: does not store every past examples
2. Intrinsically **on-line** and adapted to **non stationary environments**

But can we:

1. **Formalize** this?
2. **Measure** this advantage?
3. Turn this into a **learning strategy**?

Tracking
Analysis

A fundamental tradeoff

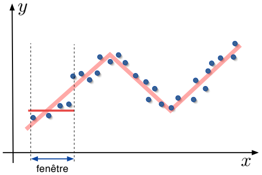
Temporal Consistency		i.i.d. data
Small memory	$\longleftrightarrow$	**Large memory**
Simple $\mathcal{H}$		**"Complex"** $\mathcal{H}$

Tracking

A new inductive problem

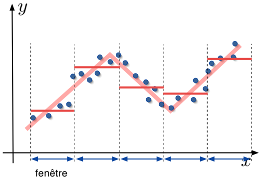
Notion of ***temporal consistency***

$f(\cdot, \theta_t)$ continuous
and with bounded variation / θ_t



New inductive criterion

$$
\begin{aligned}
L_{\langle 0,T \rangle}(r) \; &= \sum_{t=0}^{T} \ell(h_t(\boldsymbol{x}_t), y_t) \\
&\quad + \lambda \sum ||h_t - h_{t-1}||^2 \\
&\quad + \text{Capacity}(\mathcal{R})
\end{aligned}
$$



Do not optimize the choice of ONE h any longer!!

but optimize the learning rule ($r \in \mathcal{R}$) instead: $(h_{t-1}, \boldsymbol{x}_t) \xrightarrow{r} h_t$!!

Tracking

Issues in want of answers

New inductive criterion

$$L_{\langle 0,T \rangle} = \sum_{t=0}^{T} \ell(h_t(\boldsymbol{x}_t), y_t) + \underbrace{\lambda \sum ||h_t - h_{t-1}||^2 + \text{Capacity}(\mathcal{R})}_{\text{new criterion}}$$

How to find a good learning rule $r \in \mathcal{R}$?

rule complexity

(memory + complexity)

$\longleftrightarrow$

Local complexity
of target function

Control of bias-variance (overfitting)

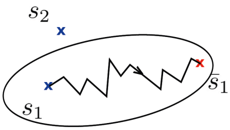
Transfert
Définition

Ré-utilisation de connaissances acquises dans un contexte (pour résoudre une tâche) dans un autre contexte (pour résoudre une autre tâche)

Éventuellement changement de domaine

Exemples :

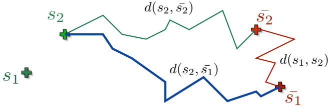
- Adaptive A* / Planification
- Règles apprises pour classer des factures utilisées pour classer des réclamations
- Analogie : $a\ b\ c \rightarrow a\ b\ d\ :\ i\ j\ k \rightarrow ?$

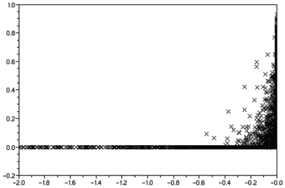


Transfert
Questions

Sous quelles conditions le transfert est avantageux ?

(et quand vaut-il mieux repartir de 0 ?)





FC08 L. Fedon et A. Cornuéjols (2008) *"Comment optimiser A* adaptatif"* Proc. of RFIA-08, 2008.

Apprentissage guidé

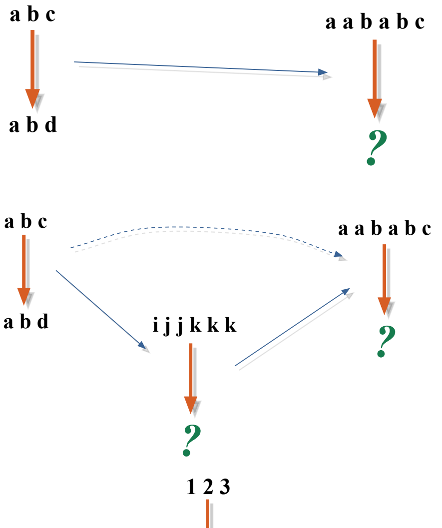
Choix des exemples effectué par un professeur

Questions

- Comment fournir **la séquence** d'exemples la plus efficace
  - pour un apprenant
  - dans un but donné

Les effets de séquences
Exemple

[Sur 24 étudiants de DEA, 1996]



- Long et difficile
- Grande variété de réponses

- Beaucoup plus rapide
- Spectre de réponses beaucoup plus serré

Questions

Effets de l'ordre

- Qu'est-ce qui les caractérise ?
  - Optimalité
  - Oubli du passé
- Comment les éliminer ?
- Comment les utiliser ?
- Que révèlent-ils sur la notion d'information transmise d'un état au suivant ?

Quelle **mémoire du passé** ? Que peut-on réutiliser ? Doit-on réutiliser ?

Nouveau critère inductif ?

Outline

Conclusions

Emerging applications **can not** be solved within the classical setting

- **non i.i.d. data**: the sequence conveys information
- Learning is a **limited rationality activity**

Lots of open questions

- ***How to deal with non i.i.d. data***
  - **What to memorize?** / What to forget?
  - How to cope with or take advantage of **ordering effects**?
  - How to **facilitate future learning**: change representations, ...?
- ***What should the inductive criterion be?***
  - How to take the **computational resources** into the inductive criterion?
  - Optimize $h \in \mathcal{H}$ or $r \in \mathcal{R}$?

Already **a growing body of works**

- Covariate shift, transduction, concept drift, tracking ...
- ***Transfer*** *between tasks* / ***Teachability***

A mature science of on-line learning is in demand

The future ...

... starts here!

THANK YOU!